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The two-thirds power law in motion perception

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If a visual motion abruptly vanishes, the vanishing point is mislocalized in the anticipated direction of the motion (cf. Freyd & Finke, 1984; Hubbard & Bharucha, 1988; Verfaillie & d'Ydewalle, 1991). Here, we replicate this effect for curvilinear motions, showing that the compatibility with human movements, as expressed by the two-thirds power law (cf. Lacquaniti, Terzuolo, & Viviani, 1983; Viviani, 2002), specifically contribute to this anticipation error. Thus, the compatibility effect does not manifest itself solely in an overshooting of the judged vanishing position in comparison to the objective vanishing position, but also in a more accurate anticipation of the curvilinearity of the forthcoming motion. The latter effect only occurred for spatially unpredictable target motions. Spatially more predictable target motions allowed for a different kind of anticipation, which overrode the compatibility effect. The results are discussed with regard to the notion of an action-related influence on motion perception.

Motion is an intriguing topic of visual perception research. On the one hand, we move our eyes, heads, and bodies even in stationary environments—thereby generating a continually changing stimulation pattern on our retinas. As suggested by Gibson (1979), these changes are regularly related to our movements, allowing us to extract invariant information from the environment. On the other hand, there are all kinds of motions in our environment, including other humans' movements. In order to respond to these motions appropriately, we often need to anticipate them. This raises the question of how these motions are represented and how such representations contribute to forming anticipations about the future course of motion.

The notion of motion anticipation presupposes that we use information about some of the motion's underlying causes. This information may be derived from the fact that all motions in the environment follow the same general laws. Thus, the visual system might have internalized some environmental invariants (cf. Shepard, 1984, 1994). In particular, it has been proposed that physical

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momentum has a representational counterpart, which manifests itself in the tendency to mislocalize the vanishing point of a motion in the motion direction (cf. Freyd, 1987; Freyd & Finke, 1984). Regarding human movements, a different kind of information seems to be available, too. In particular, the perception of human movements may be constrained by the way we would generate them (cf. Shiffrar & Freyd, 1990; Viviani, 2002). Thus, a motion that is compatible with a human movement should be better anticipated than an incompatible one. In the following, we try to show that the latter kind of anticipation specifically affects the representational momentum effect that is generally regarded as a good measure of motion anticipation (cf. also Verfaillie & Daems, 2002; Verfaillie & d'Ydewalle, 1991). Before reporting two experiments, we will give a brief overview of the main studies in the different areas of research.

Representational momentum studies have been originally motivated by the observation that single static displays, indicating a motion, are harder to discriminate from different displays of the same motion when the latter displays depict the motion in real-world than backward order (cf. Freyd, 1983; Freyd & Pantzer, 1995). These observations have been interpreted with regard to the notion of dynamic mental representations (cf. Freyd, 1987), which are mainly characterized by the assumption that time is represented in an analogue format. Thus, the representation of an external event lasts for a minimal amount of time. Consequently, it cannot be halted instantaneously. At least, this conclusion has been suggested by the following studies. Freyd and Finke (1984) presented three static rectangles at different orientations along a possible path of rotation and asked subjects to judge whether or not a fourth rectangle had the same orientation as the previous one. It turned out that this judgement was more difficult when the fourth rectangle was rotated in the direction of the implied motion rather than in the opposite direction. This localization error has been named the representational momentum (RM) effect, due to the fact that it seemingly provided evidence for an internalization of physical momentum. As a moving object usually needs some time to come to a halt, the mental representation of the motion also does. In consequence, the location of an abruptly vanishing motion will be misperceived in motion direction. In accordance with this view, the RM effect increases with the velocity of the displayed motion (cf. Freyd & Finke, 1985). Moreover, the RM effect increases during the first 300 ms after motion offset, before it is reduced again—which could be a result of a memory averaging process (cf. Freyd & Johnson, 1987).

The internalization hypothesis has been challenged by the observation that RM effects, at least for continuous motions, are mediated by smooth pursuit eye movements. In other words, when subjects fixate a stationary point, thereby inhibiting the spontaneous tendency to pursue the target motion, no RM effect can be observed at all (cf. Kerzel, 2000). It should be kept in mind, however, that smooth pursuit eye movements are also liable to anticipations, which is revealed, for instance, by the observation that the spatiotemporal distance between target and eye position is smaller than could be expected on the basis of reaction times (cf. Keating, 1991) or by the observation that smooth pursuit eye movements slow down before an anticipated direction change of the target motion (cf. Boman & Hotson, 1992).

Moreover, the anticipated direction of motion of the target need not always be in the same direction as the current direction of the motion. Thus, Hubbard and Bharucha (1988), displaying continuous motions of a single dot, showed that the localization error might even be opposite to the motion direction at the vanishing time. A moving dot bounced within a rectangle several times before it vanished just before, during, or after its collision with a wall. Only in the latter case was there a positive representational momentum effect. When the dot vanished during the collision, the sign of the representational momentum effect was even reversed. Thus, subjects apparently anticipated the direction change. In consequence, the RM effect may only reflect the time needed for the assumed anticipation process to come to a halt. Or, more generally, the RM effect may reflect the fact that an anticipation has occurred at all.

In order to investigate whether these anticipations are influenced by the way human movements are generated, we distinguished motions that are compatible with human movements from motions that are not. For this purpose, we relied on the two-thirds power law (cf. Lacquaniti et al., 1983). The two-thirds power law describes a regular relationship between the instantaneous velocity V_t and the radius of curvature R_t of planar endpoint movements of the hand–arm system. For all points sufficiently remote from inversion points—where R_t goes to infinity—the relationship can be described by $V_t = k * R_t^{ß}$. The factor k indicates the average velocity of the movement; it tends to change at points of inflection or at the junction between figural units. The exponent ß indicates the amount with which the instantaneous velocity is influenced by the radius of curvature. In the original formulation, the angular velocity and the curvature were related. The angular velocity is the quotient of the instantaneous velocity and the radius of curvature, and the curvature is the inverse of the radius of curvature. Basically, the two-thirds power law states that the instantaneous velocity is lower in more curved parts than in less curved parts of the trajectory. For human movements, this relation manifests itself in a specific way. Thus, estimates of the ß-exponent (in the commonly used formula indicated above) invariably amounts to a value of one third (this is equivalent to an exponent value of two thirds in the original formulation). This value is observed both for simple, predictable forms, such as ellipses, and for rather complex, unpredictable forms, such as scribbles. Moreover, this value also holds true for isometric drawings—as realized by a force manipulandum (cf. Massey, Lurito, Pellizzer, & Georgopoulos, 1992). Finally, subjects cannot track a target motion accurately when it deviates from the two-thirds power law. This has been demonstrated for manual movements (cf. Viviani, Campadelli, & Mounoud, 1987; Viviani &

Mounoud, 1990) as well as for smooth pursuit eye movements (cf. de'Sperati & Viviani, 1997).

Importantly, this characteristic of human motor control seems to influence motion perception. Thus, Viviani and Stucchi (1989) estimated the eccentricity at which the subjects were no longer able to discriminate whether the motion of a single dot described a vertical or a horizontal ellipse. It turned out that this estimate was biased in the direction of the eccentricity that corresponded, via the two-thirds power law, to the velocity profile of the moving dot. Correspondingly, Viviani and Stucchi (1992) showed that the perception of the velocity of a moving dot is influenced by the trajectory's eccentricity. Subjects adjusted the ß-value of the formula of the power law underlying the dot's motion in order to attain a subjectively constant velocity—the objectively constant velocity corresponds to a ß-value of 0. However, the adjusted ß-values were biased in the direction of the ß-value of human movements—in the case of scribbles, the adjusted ß-values were virtually one third. Subsequent studies investigated whether this effect depended on smooth pursuit eye movements, which was not the case (cf. de'Sperati & Stucchi, 1995), and whether comparable effects also occurred in the kinaesthetic modality, which was the case (cf. Viviani, Baud-Bovy, & Redolfi, 1997).

These observations are remarkable because a single frame does not specify a human movement at all. Moreover, the overall motion path is not specific to human movements either. Rather, the specificity of the motions that are compatible with human movements is only provided by the appropriate, time-dependent integration of the visual motion information. Accordingly, the cited studies only used unspeeded judgements. These judgements might have favoured a sampling of the motion signal over time. In particular, judgements of constant velocity seem to presuppose that velocity changes are not detected. Generally, velocity changes are, at least when they occur gradually, difficult to perceive. Their perception might involve a comparison of the motion at different points in time (cf. Gottsdanker, 1956; Schmerler, 1976; Werkhoven, Snippe, & Toet, 1992). Eccentricity, being defined by the ratio of the ellipse's two axes, refers by definition to a global property of the motion, of which the judgements presuppose an integration of the motion information at least over one period.

Finally, these studies did not explicitly address the question to which degree the two-thirds power law can be used to anticipate a motion. Moreover, everyday motion anticipations usually refer to local properties of a motion, such as the time at which a moving target will arrive at a given location. In principle, the two-thirds power law could be used for this purpose, given that the motion path is known in advance. The hypothesis that subjects actually use their knowledge about the two-thirds power law for the spatiotemporal anticipations in question is indirectly supported by the aforementioned tracking studies (cf. de'Sperati & Viviani, 1997; Viviani et al., 1987; Viviani & Mounoud, 1990). However, in these studies the anticipation could have been bound to the execution of a related pursuit movement.

Nonetheless, there is some evidence that this kind of anticipation also plays a role in seemingly unrelated tasks. Kandel, Orliaguet, and Viviani (2000b) reproduced the generation of a handwritten letter by a moving dot and asked subjects to predict which of two possible letters would come next, a task they accomplished well. However, if the kinematics of the trajectories were manipulated by changing the ß-value of the motion law, leaving its motion path virtually unchanged, the accuracy of prediction markedly dropped.

These results have been interpreted with regard to the notion of a perceptual or motor anticipation (cf. also Chaminade, Meary, Orliaguet, & Decety, 2001), which might provide a conceptual link to the aforementioned representational momentum studies. Still, there are some potentially crucial differences. Thus, representational momentum effects have been studied so far mainly with linear or circular motions, which were displayed at a constant velocity. Moreover, they used a rather low number of possible vanishing points within a rather high number of experimental trials, which might have fostered a between-trial predictability (cf. also Kerzel, 2002). In contrast, human movement paths are usually not perfectly linear or circular and the velocity is virtually never constant. Moreover, we usually do not observe the same movement again and again. Nonetheless, human movements might have some predictability, which could be derived from the way humans generate the motion.

EXPERIMENT 1

The first experiment tested whether a representational momentum effect is affected by whether or not the stimulus motion follows the two-thirds power law. For this purpose, we used motion stimuli with rather unpredictable motion paths. An example of a motion path used in the following two experiments is given in Figure 1. Moreover, the motion stimuli either complied with the two-thirds power law or deviated from it.

If a motion follows the two-thirds power law, it also shows continuous variations of velocity and radius of curvature. Because velocity changes by themselves may contribute to the predictability of the forthcoming motion path—in a beneficial or in a detrimental way—we generated two control motions according to the formula of the two-thirds power law. The first one, with a ß-value of .00, is characterized by a constant velocity. The second one, with a ß-value of .66, is characterized by pronounced velocity changes—which are more pronounced than those of motions with a ß-value of .33. If the localization errors for ß = .33 differ from the localization errors for ß = .00 and from the localization errors for ß = .66 in the same direction, velocity changes by themselves cannot be the only distinguishing factor. Instead, it is more parsimonious to assume that the compliance with the two-thirds power law is the crucial factor.

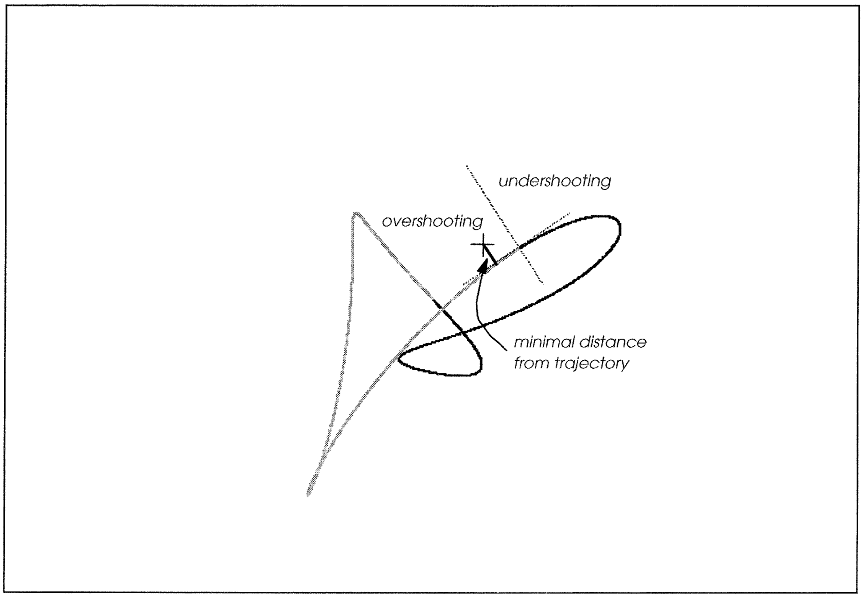

Figure 1. One of the motion paths used in the first experiment. The black line indicates the part of the motion path that was visible in the experiment. The crossing point between the black and the gray line indicates the objective vanishing point. The small cross indicates a hypothetical response. The large dotted cross indicates the coordinate frame that was used to classify the responses as overshooting or undershooting. For overshooting responses, the minimal distance of the forthcoming trajectory was determined as well. The frame indicates the borders of the screen.

Previous studies of the two-thirds power law in motion perception did not report a dependence of the visual illusions in question on the average velocity (cf. also de'Sperati & Stucchi, 1995). Thus, one may tentatively assume that effects of the two-thirds power law on motion perception are robust over a wide range of biologically plausible velocities. However, the representational momentum effect is affected by the average velocity. Moreover, it may hinge on smooth pursuit eye movements (cf. Kerzel, 2000). Inasmuch as smooth pursuit eye movements are most efficient within a certain velocity range (cf. Horii, 1994), the representational momentum effect may deteriorate for faster motions. Thus, we used two average velocities: The slower one was selected so that the target motion could be tracked easily by smooth pursuit eye movements. The other one was twice as fast. If the effect of the two-thirds power law depends on smooth pursuit eye movements, it should be more pronounced in the slow condition. If it is independent from smooth pursuit eye movements, the velocity manipulation should not matter.

The representational momentum effect is defined as a localization error in motion direction, which is indicated, for straight motion paths, by the constant error. A positive constant error is usually interpreted as evidence for motion anticipation. Assuming a normal distribution, the constant error should be systematically related to the probability that the subjective vanishing point overshoots the objective vanishing point, which can also be estimated for curved motion paths. Following the above considerations, the relative frequency of overshooting should be higher for motions that are characterized by a ß-value of .33 than for other ones. Moreover, the use of curved motion paths opens the possibility to assess the nature of overshooting responses. If the subjects anticipated the forthcoming trajectory, they should also account for the direction changes that occurred continuously in our motion stimuli—except for one point of the trajectory. Thus, the minimal distance from the forthcoming trajectory should be lower for motions that are compatible with the two-thirds power law than for incompatible ones.

Method

Participants. Twenty-four participants took part in the experiment, five of them male. They ranged in age from 19 to 40 years. All participants had normal or corrected-to-normal vision. They received payment for their participation.

Apparatus and stimuli. The visual stimuli were displayed on an Apple 17-inch monitor with a spatial resolution of 832 × 624 pixels; the vertical sync frequency was 100 Hz. Viewing distance was unrestrained at a distance of about 50 cm. The visual stimuli consisted of a black dot moving on a white background; the diameter of the dot was about 0.38 cm—which roughly corresponds to 0.44°. The room light was dimmed. Using the program GENSCRIB, we generated a set of more or less random closed trajectories with continuous first and second derivatives. The trajectories followed the generalized two-thirds power law:

$$V_t = k * \left(\frac{R_t}{1 + \alpha * R_t}\right)^{\beta} \text{(cf. Viviani \& Stucchi, 1992).}$$

Using an α-value of 0.05, we generated 40 motion paths with always one point of inflection. Three exemplars were selected for each motion path, characterized by the ß-values of .00, .33, and .66. Although all trajectories were different, they shared some features: First, the total duration of a motion was set to 6 s. Second, the motion always started and ended at the centre of the screen. Third, the maximum horizontal and vertical distance from the centre was about 11.54 cm—or 13.00°. Half of the trajectories were displayed at the double rate so that the total duration of these fast motions amounted to 3 s. The trajectories were displayed until the dot reached a preselected point. This point could occur at 490–5760 ms after motion onset, or, 245–2880 ms, respectively. In consequence, the dot never vanished at the centre of the screen. For all ß-values, the temporal positions of the offset points were the same. These temporal positions were selected so that the spatial position of the offset points only differed slightly between the different ß-values. Accordingly, the velocities at the offset points varied depending on the ß-value: For all trajectories, the endpoint velocity was highest for ß = .00 and lowest for ß = .66. Average values are reported in Table 1.

The responses were recorded with the computer mouse, steering a visual cross on the screen.

Procedure. A session comprised six blocks of 40 trials each. Within each block, each trajectory occurred once. The order in which the trajectories were displayed changed from block to block. Each block comprised a different combination of the three ß-values and the two average velocities. Whereas the average velocity alternated between successive blocks, the ß-values of the stimuli were kept constant for two blocks, respectively. Varying the order of the possible vanishing points within a pair of blocks as well as the order of the ß-values, we created twelve different block sequences. Two participants were randomly assigned to each block sequence, respectively.

Participants were instructed to localize the point where the moving dot disappeared. Each trial started with the participants moving the cursor to the centre of the screen. For this purpose, they used the computer mouse. When the mouse button was pressed at the starting point, the cross disappeared, and the moving dot emerged after the button release; 500 ms after the disappearance of the moving dot, the cross reappeared at the centre of the screen and the participants steered it to the perceived last point of the dot. No time constraints were imposed for this task. The participants were requested not to move the mouse during the motion display. If they did so nonetheless, an error message occurred at the end of the trial. Each block started with three practice trials, consistent with the stimulus condition in the subsequent block. Once again, participants started each trial on their own. Breaks of 1 minute were inserted between the blocks. The whole session lasted about 40 minutes.

TABLE 1
Average velocities at offset points in Experiment 1

Md	*ß = 0.00*	*ß = 0.33*	*ß = 0.66*
Slow condition	9.58 cms^{-1}	5.20 cm^{-1}	2.56 cms^{-1}
Fast condition	19.15 cms^{-1}	10.40 cm^{-1}	5.12 cms^{-1}

Data analysis. In order to assess whether or not a mislocalization of the vanishing point of a motion in the motion direction occurred in the present experiment, we calculated the tangent of the trajectory at its objective vanishing point and subsequently determined the relative frequency with which the subjective vanishing point overshot the perpendicular of this tangent. This relative frequency measure has the advantage that it is relatively robust with regard to slight variations of the assumed direction—thus, we can allow for the possibility that the last seen dot position precedes the objective vanishing point by a short time. Besides, the relative frequency measure is relatively robust to distributional assumptions. It only indicates whether or not an overshooting response occurred within a trial. An overshooting response need not imply that the forthcoming motion is actually anticipated, however. In particular, the forthcoming motion direction might as well be anticipated or not. Thus, we determined a second dependent measure, which is the minimal distance of the subjective vanishing point from the forthcoming trajectory. These minimal distances were averaged, for each subject, over the overshooting trials of a given condition. Finally, we restricted the analysis of these two dependent measures to error-free trials. A trial was counted as an error when the Euclidian distance between the objective and the subjective vanishing point was more than two standard deviations higher than the arithmetic mean of all Euclidian distances.

We assessed the rather specific prediction of our main hypothesis by a priori contrasts, in which the ß-value of .33 was compared with the equally weighted ß-values of .00 and .66. This rules out the possibility that the ß-value has a linear effect on localization performance—including a null effect. It should be kept in mind, however, that the localization performance for ß = .33 should also be maximal for the relative frequencies of overshooting and minimal for the minimal distances from the forthcoming trajectory. Additionally, we calculated separate contrast analyses for each velocity condition in order to attain indirect evidence for a potential moderator effect of the velocity condition on the contrast effect in question.

Results

In 187 out of 5760 trials, the Euclidian distance between the objective and the subjective vanishing point more than two standard deviations ($s = 0.64$ cm) higher than the mean ($M = 0.82$ cm) of all Euclidian distances. This roughly corresponds to an error rate of 3%. Fewer errors occurred in the slow condition (1.5%) than in the fast condition (5.0%), which was significant, $F(1, 23) = 51.46$, $p < .01$. Moreover, the number of the errors tended to increase with the ß-value, ß = .00: 2.5%; ß = .33: 3.4%; ß = .66: 3.9%, although this effect was not significant, $F(2, 46) = 2.96$, $p = .06$. There was no significant interaction between these factors ($F\ (2, 46) = 0.36$; $p = .70$).

The serial position of the block did not significantly affect either of the dependent measures. After an informal inspection of the effects of other variables within individual blocks, we decided to collapse the data over the block variable.

For the relative frequency of overshooting, the contrast between the ß-value of .33 and the other two ß-values was significant, $F(1, 23) = 7.53$, $MSE = 0.004$, $p = .01$. The subjective vanishing points more often overshot the objective vanishing points in the case of compatible motions than in the case of noncompatible ones (cf. Figure 2). For the minimal distances from the trajectory in overshooting trials, the corresponding contrast also became significant, $F(1, 23) = 14.67$, $MSE = 0.27$, $p < .01$. The subjective vanishing points were closer to the forthcoming trajectory points in the case of compatible motions than in the case of noncompatible ones (cf. Figure 3). The velocity condition had a significant effect on the relative frequencies of overshooting, $F(1, 23) = 41.49$, $MSE = 0.01$, $p < .01$, as well as on the minimal distances from the forthcoming trajectory, $(1, 23) = 126.93$, $MSE = 0.34$, $p < .01$. For the slower velocities, the relative frequencies of overshooting were higher and the minimal distances from the forthcoming trajectory were lower than for the faster velocities. Separate analyses for the two velocity conditions showed that the contrast effect on the relative frequencies of overshooting can be primarily attributed to the slower velocities: slow: $F(1, 23) = 8.10$, $MSE = 0.004$, $p = .01$; fast, $F(1, 23) = 1.67$, $MSE = 0.004$, $p = .21$. Regarding the minimal distances from the forthcoming trajectory, the contrast effect was again more pronounced in the case of the

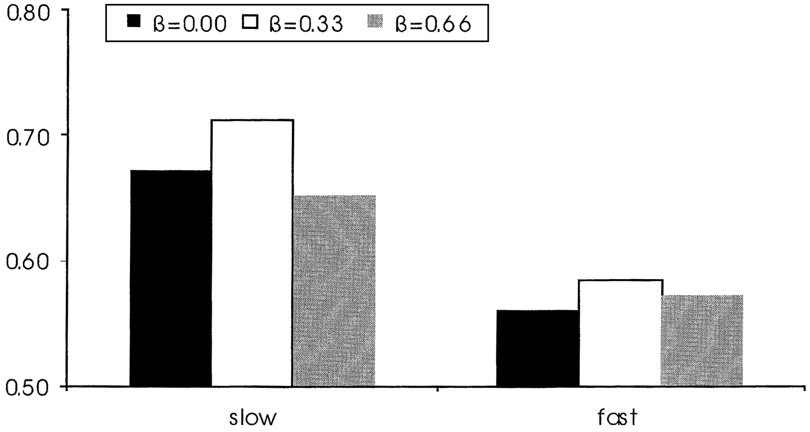

Figure 2. Relative frequencies of overshooting in Experiment 1. On the abscissa, the two velocity conditions are plotted. The black columns indicate the relative frequencies of overshooting for ß = 0.00. The white columns indicate the relative frequencies of overshooting for ß = 0.33. The grey columns indicate the relative frequencies of overshooting for ß = 0.66.

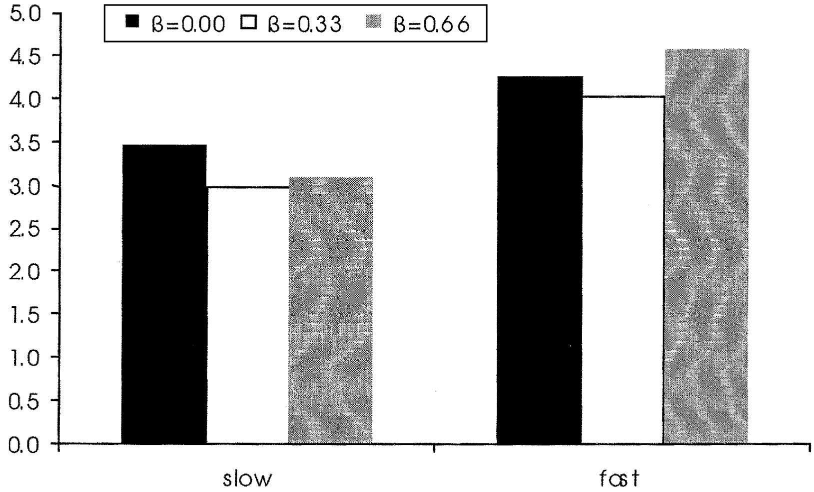

Figure 3. Minimal distances from trajectory (in cm) in overshooting trials of Experiment 1. The notation follows Figure 2.

slower velocities: slow, $F(1,23) = 7.72$, $MSE = 0.36$, $p = .01$; fast, $F(1,23) = 4.98$, $MSE = 0.27$, $p = .04$.

Discussion

Motions that are compatible with the two-thirds power law differed from incompatible motions with regard to the relative frequency of overshooting. Moreover, the overshooting responses were more accurate for compatible motions than for incompatible ones. These contrast effects were more pronounced for slower velocities. One reason to expect such a pattern of results is given by the observation that smooth pursuit eye movements work best within a certain velocity range. For target motions with higher velocities, the number of saccades increases (cf. Horii, 1994). Since saccades do not follow the two-thirds power law, the tracking performance may be less influenced by this motor constraint. Generally, the higher velocities reduced the amount of overshooting. This is surprising, because the constant error generally increases with velocity in representational momentum studies (cf. Hubbard, 1995).

One difference between this study and most other representational momentum studies is given by the fact that the instantaneous velocity of the target motion varied continuously (see also Finke, Freyd, & Shyi, 1986). Particularly, the vanishing points were always located within deceleration phases of the motion. Thus, the amount of overshooting may be affected by two factors: The velocity and the deceleration at the time the dot vanished. Whereas a higher velocity leads to more overshooting, a higher deceleration presumably has the opposite effect, i.e., the pursuit motion sooner comes to a halt. It is important to note that doubling the velocity amounts to quadrupling the deceleration. Thus, the net effect of overshooting may become smaller. Alternatively, one may assume that the lower velocity gives the observers more time to anticipate the motion, which may manifest itself in a more pronounced overshooting. At least, this is the suggested interpretation of Wexler and Klam (2001), who recently reported a similar effect of speed on motion extrapolation. Interestingly, their motion displays reproduced human movements.

Specifying a motion by the two-thirds power law inevitably establishes a continuous variation of the instantaneous velocity, the radius of curvature and other correlated parameters. Thus, one may wonder whether the observed effect of the ß-value was mediated by any of these parameters. Specifically, we tested the assumption that a ß-value of .33 differs from the other ß-values within contrasts that weighted the ß-values of .00 and .66 equally. Thus, we excluded the assumption that the ß-values had a linear effect on the localization performance. However, if the localization performance actually depended on the velocity at the vanishing point, a nonlinear effect might also arise. In this case, the ß-values should have a monotonic effect on the localization performance. This was not the case, as indicated by the maximum of the relative frequency of overshooting and the minimum of the minimal distance from the forthcoming trajectory at the intermediate ß-value of .33. However, it is conceivable that the nonmonotonic effect of the ß-values on the localization performance resulted from a summation of opposite, nonlinear effects of velocity and deceleration. In this case, a change in the ratio of these parameters, which was introduced by the doubling of the velocities, should have a thorough effect on the result pattern. However, this was also not the case.

EXPERIMENT 2

The primary aim of the second experiment consisted in the generalization of the foregoing results. For this purpose, we used a different set of motion stimuli. This allowed us to assess the interplay of different kinds of anticipation. In the preceding experiment, we used minimally predictable motion paths in order to maximize the relative predictability provided by the covariation of instantaneous velocity and radius of curvature. However, other properties of the trajectory may also provide predictive cues. In particular, the regular relationship between the duration, the length, and the maximum velocity of the movement segment, which regularly occurs halfway through the motion segments in question, may be used to anticipate the segment's endpoint (cf. Edelman & Flash, 1987; Kandel, Orliaguet, & Boe, 2000a). In order to assess the relative importance of the compatibility with the two-thirds power law in comparison with the endpoint anticipations, we chose motion stimuli that allowed for some anticipation of segment endpoints. In particular, we varied the relative spatial and temporal distance of the vanishing points from the segment endpoints on the basis of the assumption that the probability of the endpoint's anticipation increases as the relative distance decreases.

In order to enable these anticipation as outlined above, we used motion paths that were composed of pairs of identical segments, i.e., half ellipses. The vanishing point always occurred in the second segment of a pair. In order to prevent subjects from switching to a spatial strategy for their localization judgements, the identical segments were always oriented in opposite directions—consequently, each pair resembled a sine wave. Moreover, the motion paths were composed of four out of five possible pairs. This nonidentical repetition was introduced in order to allow for the possibility that subjects estimate the average movement duration or velocity on the basis of preceding segments. Thus, we hypothesized that an endpoint anticipation only builds up gradually during the perception of a repetitive motion. An example of a motion path used is given in Figure 4.

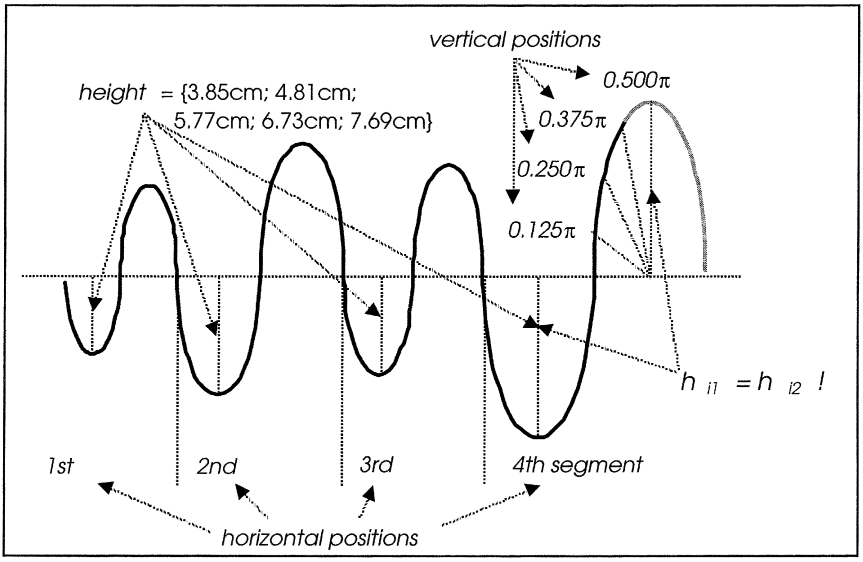

Figure 4. One of the motion paths used in Experiment 2. The black line indicates the part of the motion path that was visible in the experiment. The crossing point between the black and the gray line indicates the objective vanishing point. The text as well as the dotted lines were not visible in the experiment. They are just shown in order to illustrate the different stimulus variables used in the present experiment. The frame indicates the borders of the screen.

Method

Participants. Twenty-four new participants took part in the experiment, nine of them male. They ranged in age from 19 to 36 years. All participants had normal or corrected-to-normal vision. They received payment for their participation.

Apparatus and stimuli. The same apparatus as in the previous experiment was used. Viewing distance was unrestrained at a distance of about 40 cm. The room light was dimmed. The stimulus consisted of a black dot, moving in front of a white screen. The diameter of the dot was 0.38 cm, which roughly corresponds to 0.54°. The trajectory consisted of 2×4 half ellipses, characterized by an eccentricity of 0.96. This amounts to a rather low radius of curvature at the highest or lowest points of the trajectory and a rather high radius of curvature in between. The elliptic trajectory approaches an inversion point, enabling a relatively smooth concatenation of segments of different lengths. Four upper halves alternated with four lower halves of ellipses so that the resulting trajectory resembled a wave. In Condition A, the trajectory started with an upward segment; in Condition B, it started with a downward segment. All trajectories started on the left side of the screen. The upper segments all had different heights—and, therefore, different lengths; the same applies to the lower halves. Four levels of height occurred within a trajectory. Each level occurred twice: Two subsequent segments always had the same height. In all, five heights were used in this experiment, ranging from 3.85 cm to 7.69 cm—or from 5.50° to 10.88°, respectively. Correspondingly, the length of the segments from their starting points to the points of minimum radius of curvature—i.e., the highest or the lowest point—varied between 4.18 cm and 8.35 cm—or between 5.97° and 11.79°, respectively. The dot always disappeared in the second half of a pair of half ellipses. Describing the positions of a half ellipse by its passed angles so that its start has a value of 0 and its end a value of π (rad), the dot could vanish at the $0.125\,\pi$, the $0.250\,\pi$, the $0.375\,\pi$, or the $0.500\,\pi$ position. Of course, the $0.500\,\pi$ position is the position of the minimum radius of curvature; the $0.250\,\pi$ position is halfway between the position of the minimum radius of curvature and the start position; and the other two positions are halfway between the former position and the start position or the position of the minimum radius of curvature. The respective velocities, averaged over the different absolute heights of the half ellipses, are reported in Table 2.

Procedure. A session comprised three blocks of 80 trials each. Within each block, each combination of the four vertical positions, the five heights, and the four horizontal positions of the pairs of the half ellipses occurred once in a random order. This order was kept constant over all blocks and participants.

TABLE 2
Average velocities at offset points in Experiment 2

Md	*ß = 0.00*	*ß = 0.33*	*ß = 0.66*
0.500 π	7.50 cms^{-1}	3.15 cms^{-1}	1.54 cms^{-1}
0.375 π	7.50 cms^{-1}	5.31 cms^{-1}	4.43 cms^{-1}
0.250 π	7.50 cms^{-1}	8.30 cms^{-1}	10.79 cms^{-1}
0.125 π	7.50 cms^{-1}	10.37 cms^{-1}	16.97 cms^{-1}

Each block was characterized by one ß-value. The order in which the ß-values were assigned to the number of the block was balanced across the participants—the participants being assigned randomly to the six different permutations. Moreover, the participants were assigned randomly to Condition A or Condition B. The same task as in Experiment 1 was used. The only difference was that the starting point of the cursor was now located at the bottom of the screen—equidistant from its left and right border. The whole session lasted about 35 minutes.

Data analysis. The same dependent measures as in Experiment 1 were used. Besides the ß-value, the vertical position was analysed as a within-subject variable.

Results

A trial was counted as an error when the absolute error was more than two standard deviations ($s = 0.82$ cm) higher than the average ($M = 0.70$ cm) of all errors. This was the case for 75 out of 5760 trials—which roughly corresponds to 1% errors. The number of errors was lower for compatible motions (1%) than for noncompatible ones, ß = .00: 2%; ß = .66: 2%, which was significant, $F(1, 23) = 11.11$, $p = .01$. Moreover, a significant interaction occurred between the vertical position and the ß-value, $F(6, 138) = 4.30$, $p < .01$, which could be traced back to a high number of errors for $ß = .66$ at the 0.125 π position (4%). No other effect approached significance.

Preliminary analyses of the serial position of the block revealed no interactions with the ß-value so that we once more collapsed the data over this variable. We also collapsed the data over Conditions A and B as well as over the four horizontal positions, because these variables did not significantly affect the results.

For the relative frequency of overshooting, the contrast between the ß-value of .33 and the other two ß-values was significant, $F(1, 23) = 4.50$, $MSE = 0.05$, $p = .04$. The subjective vanishing points more often overshot the objective vanishing points in the case of compatible motions than in the case of noncompatible ones (cf. Figure 5). For the minimal distances from the forthcoming trajectory, the corresponding contrast was not significant any more, $F(1,23) = 2.39$, $MSE = 0.58$, $p = .13$. However, the minimal distance from the forthcoming trajectory was still smaller for compatible motions than for noncompatible ones (cf. Figure 6).

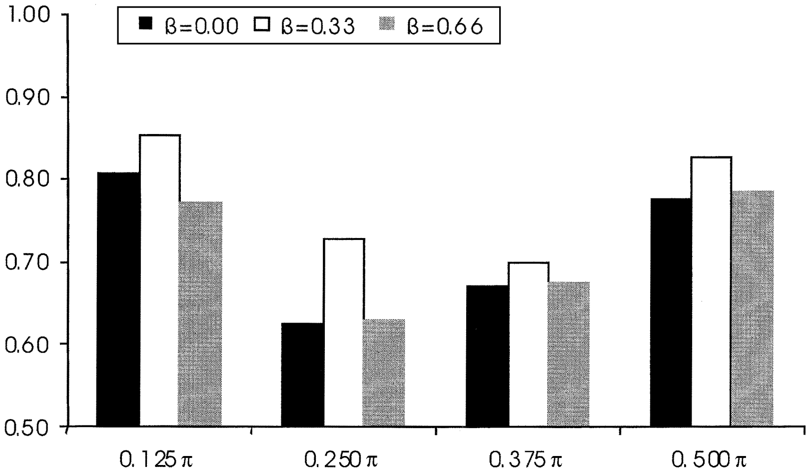

Figure 5. Relative frequencies of overshooting in Experiment 2. On the abscissa, the vertical positions of the vanishing point are indicated. The other notation follows Figure 2.

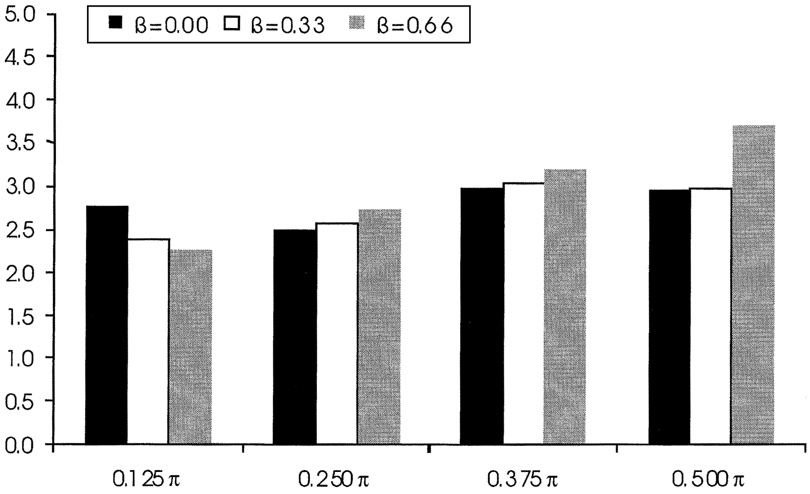

Figure 6. Minimal distances from trajectory (in cm) in overshooting trials of Experiment 2. The notation follows Figure 2.

The vertical position of the vanishing point had a significant effect on the relative frequencies of overshooting, $F(3, 69) = 8.29$, $MSE = 0.05$, $p < .01$, as well as on the minimal distances from the forthcoming trajectory, $F(3, 69) = 10.09$, $MSE = 1.13$, $p < .01$. The relative frequency of overshooting was higher for the $0.125\,\pi$ and the $0.500\,\pi$ position than for the $0.250\,\pi$ and the $0.375\,\pi$ position. The minimal distance from the forthcoming trajectory was higher for the $0.375\,\pi$ and the $0.500\,\pi$ position than for the $0.125\,\pi$ and the $0.250\,\pi$ position. Separate analyses for the four vertical positions showed that the contrast effect on the relative frequencies of overshooting was only present for the first two positions: $0.125\,\pi$, $F(1, 23) = 5.00$, $MSE = 0.01$, $p = .03$; $0.250\,\pi$, $F(1, 23) = 8.97$, $MSE = 0.02$, $p < .01$, but not for the last two positions: $0.375\,\pi$, $F(1, 23) = 0.43$, $MSE = 0.03$, $p = .52$; $0.500\,\pi$, $F(1, 23) = 1.23$, $MSE = 0.03$, $p = .27$. Regarding the minimal distances from the forthcoming trajectory, the individual contrasts all turned out not to be significant: $0.125\,\pi$, $F(1, 23) = 1.90$, $MSE = 0.39$, $p = .18$; $0.250\,\pi$, $F(1, 23) = 0.52$, $MSE = 0.49$, $p = .48$; $0.375\,\pi$, $F(1, 23) = 0.25$, $MSE = 0.41$, $p = .62$; $0.500\,\pi$, $F(1, 23) = 0.90$, $MSE = 0.52$, $p = .35$.

Discussion

Corresponding with Experiment 1, the contrast effect on the relative frequencies of overshooting showed that motions complying with the two-thirds power law are endowed with a specific predictability that may manifest itself in a higher frequency of overshooting responses. However, the contrast effect was restricted to vertical positions sufficiently remote from the point of minimum radius of curvature. Because the instantaneous velocity decreased with decreasing distance to the point of minimum radius of curvature, one may assume that there is a minimal velocity for the contrast effect to occur.

Alternatively, one may assume that the relative contribution of other factors begin to accrue with the decreasing distance to the point of minimum radius of curvature. One of these factors may be the anticipation of the segment's endpoint. This is suggested by the observation that the overall level of the relative frequencies of overshooting increased at the point of minimum radius of curvature—and, therefore, at the point of minimum velocity. The assumption that additional factors come into play is also supported by the fact that the overall level of the relative frequencies of overshooting was higher in this experiment than in the preceding one (Experiment 1: 0.63; Experiment 2: 0.74), $t(46) = 2.81$, $p < .01$.

In contrast to Experiment 1, the contrast effect on the minimal distances from the forthcoming trajectory was not significant. One cause may be the higher predictability of the motion direction, which restricted the range of possible deviations from the forthcoming trajectory. Near the midline, the motion paths approximated a straight line, which may have fostered the use of a linear extrapolation strategy—thereby disregarding local variations of the radius of curvature. This interpretation is consistent with the observation that the overall level of the minimal distances from the forthcoming trajectory increased with decreasing radius of curvature and, therefore, with increasing velocity.[1] The assumption that additional factors come into play is also supported by the fact that the overall level of the minimal distances from the forthcoming trajectory was lower in this experiment than in the preceding one (Experiment 1: 3.75 cm or 4.29°; Experiment 2: 2.90 cm or 4.15°), $t(46) = 4.27$, $p < .01$.

Assuming that different factors did contribute to the localization performance at hand, we still have to account for their interaction. The results suggest that endpoint anticipations can dominate motion anticipations on the basis of the two-thirds power law. This dominance is balanced, however, by the selectivity of the anticipated events. Whereas anticipations on the basis of the two-thirds power law are principally effective over the whole trajectory, endpoint anticipations can only be effective for circumscribed motion events. Thus, the latter anticipations cannot replace the former ones.

GENERAL DISCUSSION

This study aimed at linking two lines of investigation into visual motion perception. One line of investigation has been concerned with the tendency to mislocalize the vanishing point of a visual motion in motion direction, which has been originally interpreted in terms of a representational momentum (cf. Freyd & Finke, 1984; Hubbard & Bharucha, 1988). Subsequent studies then indicated that the anticipatory mechanisms underlying this localization error might be more general than suggested by the original formulation (cf. Verfaillie & d'Ydewalle, 1991). Here, we showed that the predictability, inherent in the way human movements are generated, can also contribute to this anticipatory error. Doing this, we extend previous findings of representational momentum effects in curvilinear motions (cf. Freyd & Jones, 1994; Hubbard, 1996). Moreover, the

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[1] As noted before, the motion stimuli of the different ß-conditions slightly differed with regard to their endpoint velocities. In standard representational momentum tasks, the (endpoint) velocity has turned out to be effective in influencing the mislocalization error in motion direction (cf. Finke et al., 1986). In order to test whether or not the endpoint velocities affected the localization judgements in our experiments, we regressed the two dependent variables on the endpoint velocities of the individual stimuli of Experiment 1, separately for the fast and the slow condition, as well as on the endpoint velocities of the individual stimuli of Experiment 2 (in order to attain a sufficient number of observations per condition, we collapsed the data over the individual subjects). None of the slope estimates of the linear regression functions turned out to be significant, with the exception of the regression function for the minimal distances from the forthcoming trajectory in Experiment 2, ß = 0.33: $t(78) = 3.79$, $p < .01$; ß = 0.66: $t(78) = 6.71$, $p < .01$. The regression functions $d_{est.} = 0.08 * V_{endpoint} + 2.19$ cm and $d_{est.} = 0.08 * V_{endpoint} + 2.31$ cm explained 16% and 31% of the variance, respectively. This finding indicates that the minimal distances from the forthcoming trajectory decreased with decreasing (endpoint) velocity. This is consistent with the assumption of a linear extrapolation strategy.

motion stimuli we used were not only more or less spatially unpredictable, but also changed from trial to trial. Thus, we tentatively conclude that a representational momentum effect need not depend on the build-up of spatial expectancies over a sequence of trials (cf. also Kerzel, 2002).

Finally, we showed that the compatibility with the way human movements are generated provide a specific contribution to the anticipation error in question. Doing this, we followed some studies showing that visual motions that are compatible with human movements are perceived or, at least, judged differently than incompatible motions, which have been interpreted as evidence for the assumption that motor processes or representations influence visual motion perception (cf. Viviani, 2002).

Compatible motions also differ from incompatible ones with regard to their higher familiarity. Thus, the compatibility with human movements might endow motions with a specific predictability, which manifests itself in an increased representational momentum effect. Of course, there are different sources of predictability, of which some might be easier to access than the one we were considering here. Thus, allowing for more ubiquitous anticipations, as given by a higher salience of segment endpoints, reduced the differences between compatible and incompatible motions with regard to the overall frequency of overshooting, although the overall level of frequency of overshooting increased.

It is this result pattern that lets us think that implicit knowledge about the way human movements are generated only comes into play when the stimulus information as well as the more ubiquitous anticipation mechanisms (cf. Müsseler, Stork, & Kerzel, 2002) do not unambiguously determine the correct response (cf. Viviani, 2002). Thus, the fact that the continuous motion vanishes unexpectedly might be a crucial condition for our action-related anticipation effect to occur. In line with this reasoning, Jordan, Stork, Knuf, Kerzel, and Müsseler (2002) showed that prior knowledge about the vanishing point, as given by the intention to stop the motion at a particular spatiotemporal position, reduced their representational momentum effect.

Interpreting our findings in terms of an influence of action-related knowledge on motion perception, we have to predict that a visual motion, generated by the subjects themselves, will also lead to anticipatory localization errors. Some evidence for this claim has already been provided by Wexler and Klam (2001). Crucially, this experimental set-up might be used to disentangle the effects of perceptual familiarity and the effects of a motor contribution to motion perception (cf. also Pellizzer, Richter, & Georgopolous, 1998). Thereby, it might also be determined whether or not this motor coactivation can be dissociated from pursuit eye movements (cf. de'Sperati & Viviani, 1997). Given that the effects of the two-thirds power law on motion perception do not always rely on concurrent eye movements (cf. de'Sperati & Stucchi, 1995), the research project started here might also be of interest to current discussions of the representational momentum effect (cf. Thornton & Hubbard, 2002).

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