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ORIGINAL ARTICLE

The dynamics of search, impasse, and representational change provide a coherent explanation of difficulty in the nine-dot problem

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Abstract The nine-dot problem is often used to demonstrate and explain mental impasse, creativity, and out of the box thinking. The present study investigated the interplay of a restricted initial search space, the likelihood of invoking a representational change, and the subsequent constraining of an unrestricted search space. In three experimental conditions, participants worked on different versions of the nine-dot problem that hinted at removing particular sources of difficulty from the standard problem. The hints were incremental such that the first suggested a possible route for a solution attempt; the second additionally indicated the dot at which lines meet on the solution path; and the final condition also provided non-dot locations that appear in the solution path. The results showed that in the experimental conditions, representational change is encountered more quickly and problems are solved more often than for the control group. We propose a cognitive model that focuses on general problem-solving heuristics and representational change to explain problem difficulty.

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Introduction

There is growing evidence that an insight to the solution of a problem can be characterized by a representational change (Knoblich, Ohlsson, Haider, & Rhenius, 1999; Ohlsson, 1984a, 1992; Öllinger, Jones, & Knoblich, 2008; Thevenot & Oakhill, 2008). This evidence makes it difficult to explain insight problem solving within the classical information-processing account (Newell & Simon, 1972), where problem solving is understood as search within a well-defined problem space (problem representation). The problem space account has no mechanism to implement a representational change for instances when the current search gets stuck, is insufficient, or does not reduce the distance to the desired goal.

There are a few accounts that attempt to remedy this omission. One suggestion is to claim that insight problems are nothing special and therefore representational change plays only a marginal role. For such explanations, problem difficulty relates either to the size of the problem space being overly large and preventing exhaustive search (Kaplan & Simon, 1990), or that problem solvers apply inappropriate heuristics when searching the problem space (MacGregor, Ormerod, & Chronicle, 2001; Ormerod, MacGregor, & Chronicle, 2002). Both accounts miss a cognitive process that addresses the change of the search space. In the first, an additional process is necessary that re-focuses on particular areas of the search space by changing the problem representation; in the second an additional cognitive process is required that changes the search space after repeated failures of the problem-solving process.

Ohlsson (Knoblich et al., 1999; Ohlsson, 1984a, b, 1992) provided a detailed framework that stressed the importance of a representational change for insight problem solving, and identified impasse as a crucial precondition. Moreover, Ohlsson (1992) identified at least three different processes that drive a representational change and thereby break an impasse: elaboration, re-encoding, and constraint relaxation. Although Ohlsson's model (1992, p. 20) incorporates search as a necessary process before an impasse is met, the framework mainly focuses on what processes occur to release the problem solver from impasse, without a great deal of elaboration on the particular nature of search processes before and after an impasse. Jones (2003) provided a model that elaborated Ohlsson's notion that insight problem solving can be understood as a sequential process of different phases (search–impasse–insight–search, see also Ash & Wiley, 2006). This model suggests that there might be a concerted interplay of search and impasse, with each affecting the other. The goal of the present study is to apply an extension of this model (see Fig. 1) to the famous nine-dot problem.

Our general proposal is that insight problem solving is a dynamic search process that proceeds in consecutive stages. In line with Ohlsson (1992) we assume that perceptual processes and prior knowledge define what is and is not represented in the initial problem representation. This representation is searched and constrained by heuristics (e.g. MacGregor et al., 2001; Newell & Simon, 1972). The search could be either successful, at which point the search terminates and a solution is found, or the search can lead to repeated failures and an impasse is reached (Ohlsson, 1992). The smaller the search space is, the faster the realization that no further progress is possible (Kaplan & Simon, 1990; Ormerod et al., 2002). If impasse was caused by the problem representation being inadequate, then it must be overcome by a representational change. The likelihood of achieving representational change is largely governed by the difficulty in relaxing self-imposed prior knowledge constraints that have been placed on the problem or in decomposing problem elements into their constituent parts (see Knoblich et al., 1999 for more information). Should representational change be achieved,[1] a new problem representation is established that subsequently changes the problem space (to be smaller or larger, ordinarily). Once again, heuristics are necessary that efficiently search of the modified problem space.

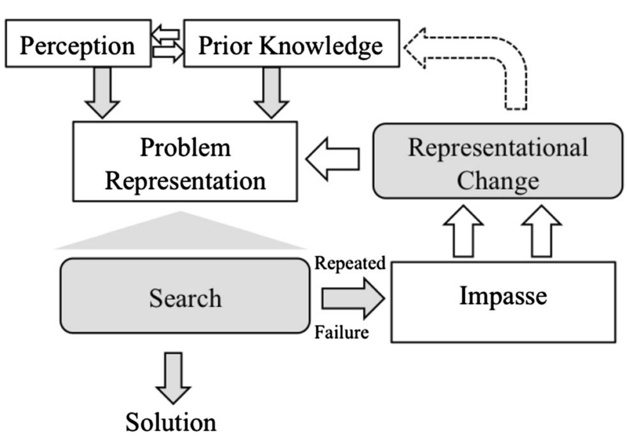

Fig. 1 Flow-chart of insight problem solving. Starting in the *top left corner*, a problem representation is established by prior knowledge and perceptual aspects, e.g. Gestalt laws, grouping, chunking. The problem representation is searched by heuristics. This can be successful: a solution is attained; or unsuccessful: an impasse is encountered. To overcome an impasse, a change of the problem representation is likely to be necessary. A new search will then be applied to the changed problem representation

The model is an elaboration of Ohlsson's representational change theory and has similar implication: First, insight is caused by a representational change. Second, there is no particular class of insight problems that necessarily requires a representational change; each problem can be solved without insight if the initial problem representation is adequate and the appropriate heuristics are available. Third, the difficulty related to attaining insight can have different causes (Kershaw, Flynn, & Gordon, 2013; Kershaw & Ohlsson, 2004). Whereas previous models have highlighted the role of perception, memory, and heuristics, the present model tries to capture the interplay between these factors. Doing so, it acknowledges that for each problem there can be different combinations of causes of problem difficulty that have to be considered. For instance, Jones (2003) demonstrated for the car park problem that heuristics play an important role before an impasse, but breaking the impasse required a representational change. Recently, Öllinger, Jones, Faber and Knoblich (2012) demonstrated that for different versions of the eight-coin problem (Ormerod et al., 2002) the main source of problem difficulty is determined by the required representational change. Nevertheless, although heuristics had no overall impact on solution rates, they still predicted the selection of coins.

In the present study, we applied the above model to the nine-dot problem (Maier, 1930) that has kept problem solving researchers busy over the last decades (Burnham & Davis, 1969; Chronicle, Ormerod, & MacGregor, 2001; Kershaw & Ohlsson, 2004; Lung & Dominowski, 1985; MacGregor et al., 2001; Maier, 1930; Scheerer, 1963; Weisberg & Alba, 1981). There is clear evidence for multiple causes of problem difficulty in this problem (Kershaw & Ohlsson, 2004), and it thus provides an opportunity to better understand the interplay of search and

[1] It does not follow that reaching impasse on a problem automatically produces a representational change (i.e. insight), since this is not always the case (Jones, 2003). Equally, whether insight is achieved and if so how quickly it is achieved depends on multiple factors, such as the difficulty of a change to the problem representation and how large the problem space is (Knoblich et al., 1999; Kershaw et al., 2013).

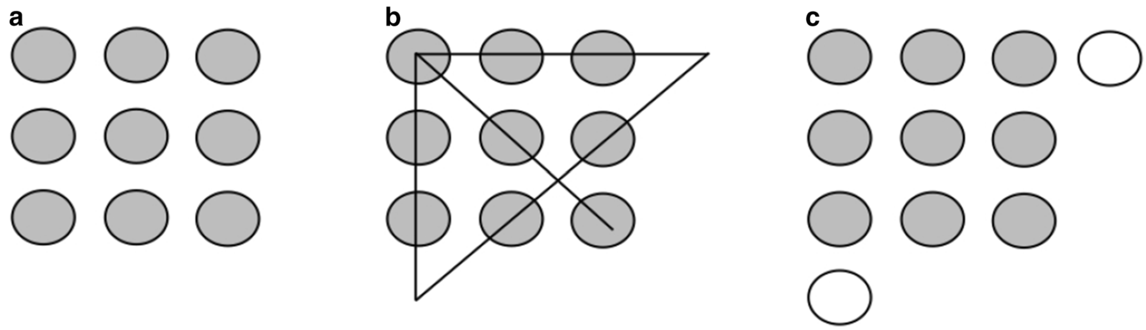

Fig. 2 The nine-dot problem (**a**) and its solution (**b**), and the 11-dot variant introduced by Ormerod et al. 1997 (**c**)

representational change. In the nine-dot problem, solvers need to connect nine dots that are arranged in a 3 × 3 square with four straight lines without lifting the pen off the paper (see Fig. 2a, b). It has consistently been shown that giving only a few minutes of time, the problem is extremely difficult to solve (see Kershaw and Ohlsson, 2004).

Explaining the difficulty of the nine-dot problem

According to the model depicted in Fig. 1, one source of difficulty in the nine-dot problem is that problem solvers initially only consider moves that remain within the 3 × 3 grid (due to a perceptually driven *boundary constraint* that keeps lines within the perceived 3 × 3 square). That is, the problem solver is working within a problem space that is *overly constrained*, but still too large to be exhaustively searched (Burnham & Davis, 1969; MacGregor et al., 2001). After repeated failure within the overly constrained problem space, the task for the problem solver is to accomplish a representational change—thus, overcoming the boundary constraint (Ohlsson, 1992; Kershaw & Ohlsson, 2004). However, relaxing the boundary constraint now leads to a problem space that is much *too large*, because if lines can begin and/or end, or turn at non-dot locations then there are a potentially infinite number of lines that can be drawn (see also Kaplan & Simon, 1990). We believe that this is why previous research has found that even when the boundary constraint is relaxed (Weisberg & Alba, 1981), participants still find that solution to the nine-dot problem is evasive unless they are given further information that may constrain the problem space (e.g. specification of non-dot locations or specification of the configuration of the solution path).

Consequently, we investigated the extent to which perceptual hints that relate to the solution path in the nine-dot problem facilitate its solution. In particular, we examined the effects of perceptual information both pre-impasse and post-impasse—first, the amount of (perceptual) solution path information that is needed to restrict the initial search space to facilitate the problem solver reaching impasse and to overcome the boundary constraint; and second, how much information is needed to constrain the subsequent post-insight problem space in order to solve the nine-dot problem. Hitherto perceptual hints have provided minimal benefit in increasing the solution rate of the nine-dot problem (Weisberg & Alba, 1981; Lung & Dominowski, 1985; Chronicle et al. 2001; Kershaw & Ohlsson, 2004). For example, providing explicit hints to draw lines beyond the virtual nine-dot square (Burnham & Davis, 1969; Weisberg & Alba, 1981) and providing explicit perceptual hints that indicate that lines go beyond the virtual square (Chronicle, et al., 2001) had limited influence on solution rates. Even when two additional dots in different colours were provided at the non-dot locations, there was little effect (Ormerod, Chronicle, & MacGregor, 1997; see Fig. 2c). Apparently, the different colour of the dots made it difficult to integrate the dots into the solution, and more importantly, as Kershaw and Ohlsson (2004) showed, it is still difficult to realize that a change in the direction of a line (i.e. a non-dot turn) is necessary at non-dot locations. That is, there are two aspects that have to be taken into account. First, achieving a representational change to overcome the boundary constraint and thereby drawing lines to non-dot locations, resulting in a relaxed search space; and second, using efficient heuristics that guide the search process in making non-dot turns within the relaxed search space.

Our model suggests that—in line with Kershaw and Ohlsson (2004)—the nine-dot problem has multiple sources of difficulty. The key reason that perceptual hints have thus far proved ineffective in the nine-dot problem is because the post-impasse search space is too large to navigate successfully within a limited time period. By using appropriate perceptual information to guide the problem solver, we hope to demonstrate how and why perceptual hints can direct attention and increase solution rates dramatically. For instance, guiding attention to the crucial problem elements significantly increases the solution rates of insight problems like Duncker's (1945) tumour problem (Grant & Spivey, 2003; Litchfield & Ball, 2011; Thomas & Lleras, 2007).

In three different conditions we systematically increased the salience of perceptual features of the nine-dot problem. The first two experimental conditions were in line with the work of Kaplan and Simon (1990) who showed that increasing the perceptual salience of crucial features can facilitate the solution of insight problems, because it helps in applying the appropriate heuristics that restrict the given problem space. The intention was to induce an impasse more quickly by (a) constraining the initial problem space by illustrating a possible solution path by presenting arrows that point to locations outside of the 3×3 grid (thus suggesting that the boundary constraint needs to be relaxed); and (b) after relaxing the constraint the arrows restrict the larger search space and indicate potential non-dot turns. In the first condition, which we label the P condition (P = path), within each dot we embedded an arrow that indicated the direction of one of the possible solutions (see Fig. 3). In the second condition, we added the particular spatial pattern of the solution (Kershaw & Ohlsson, 2004). The solution looks like an isosceles triangle (see Fig. 2b) where three lines meet at the apex. We increased the salience of a particular apex dot by indicating three arrows that meet in this dot. This condition was termed the Path-Apex condition (PA, see Fig. 3b). Our main predictions were that solution rates would be higher (PA > P > Control Group) and that participants would more quickly overcome the boundary constraint when more perceptual hints are available (PA > P > Control Group).

A further aim was to explicitly test Kershaw and Ohlsson's (2004; Ohlsson, 1992) assumption of the necessity of a representational change. Highlighting non-dot positions in the PAN (path/apex/non-dot turn, see Fig. 3c) condition should further increase the likelihood to overcome the boundary constraint. The non-dot positions explicitly draw attention to visual–spatial positions outside the imposed virtual square of the nine dots (Grant & Spivey, 2003), and should help to generate a representational change. Specifying the non-dot points should also facilitate the solution of the nine-dot problem by constraining the post-insight problem space, because by combining the arrow information with the non-dot information, the location of the non-dot turns is given. Accordingly, we predicted the participants in the PAN condition should have the best chance to quickly overcome the boundary constraint and to quickly solve the problem.

A further prediction was that for participants overcoming the boundary constraint in the control and experimental conditions, solution rates should be higher for the experimental conditions than the control group because the perceptual hints in the P, PA, and PAN condition should restrict the search space after the representational change.

Method

Participants

The 136 paid participants (32 males, mean age 25, range 18–34) were recruited by advertising at the University of Munich and in local newspapers and were randomly assigned to one of the four groups (34 per group). Nineteen additional participants were excluded beforehand because they reported to be familiar with the nine-dot problem. The data of one participant in the PAN condition was not analyzable and was therefore discarded.

Materials and procedure

Participants received a booklet that contained written instructions and five pages displaying copies of the problem (so that they could start over after failed solution attempts). Each dot in the problem statement was printed in

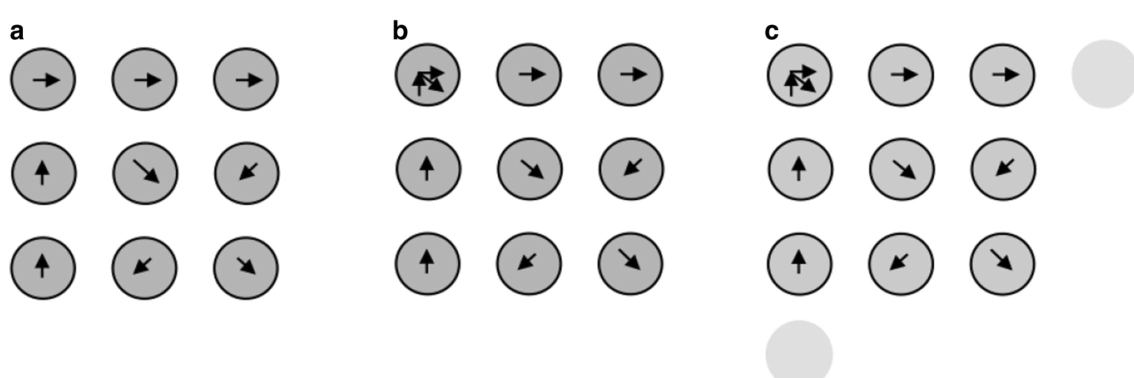

Fig. 3 **a** P condition where solution path is indicated by *arrows*. **b** PA condition with additional information regarding the location where lines meet. **c** PAN condition where additional non-dot points were provided

gray with a surrounding black circle. Each participant was tested individually in a quiet room after reading the following instruction (in German): "Connect the nine dots drawing four connected straight lines. It is not allowed to lift the pencil from the paper and it is not allowed to retrace lines. Feel free to start over as often as you like. You have 10 min for the solution. Please indicate the sequence of lines drawn using the numbers 1 to 4—1 for the first line, 2 for the second, etc."

For all experimental conditions, the dots contained arrows as displayed in Fig. 3a–c. In addition, the PAN condition consisted of two additional dots ("non-dots"), printed in a brighter gray and without a surrounding circle to distinguish them from those of the nine-dot problem (see Fig. 3c).

Data analyses

The following definitions and classifications were used for analysing the data:

- *Move*: A *move* was defined as one straight line that connected dots and/or non-dots.
- *Dot moves and non-dot moves*: Moves were classified as dot moves (a line starting and ending on one of the nine dots) or as *non-dot moves* (a line starting and/or ending at a non-dot point).
- *Solution*: A s*olution* was defined as a sequence of four moves that cancelled out all nine dots.

Results

The results are divided into three sets of analyses. As a manipulation check, we first analysed whether the perceptual hints indicating the solution path affected the move selections of participants. Second, we examined solution rates in order to assess whether and how perceptual hints facilitated the problem solution. Third, we determined how the perceptual hints influenced insight. This was achieved by (a) examining the pre-insight influence of the hints (i.e. before realizing that the boundary constraint needs to be overcome) as the number of moves required before the first move was made that went outside of the virtual square formed by the dots; and (b) examining the post-insight influence of the hints (i.e. after overcoming the boundary constraint) by analysing the solution rates for participants who achieved insight.

Manipulation check

We tested whether participants in the experimental conditions preferred moves that followed the direction of the arrows compared to the control group where the problem statement did not contain any directional information. To do so we examined for each individual the percentage of moves that followed the arrowed pattern depicted in the experimental conditions (for the control group, we analysed moves that involved the same dots in the arrowed pattern as a baseline comparison). Additionally, we assessed whether the pattern of solvers and non-solvers differed across the groups.

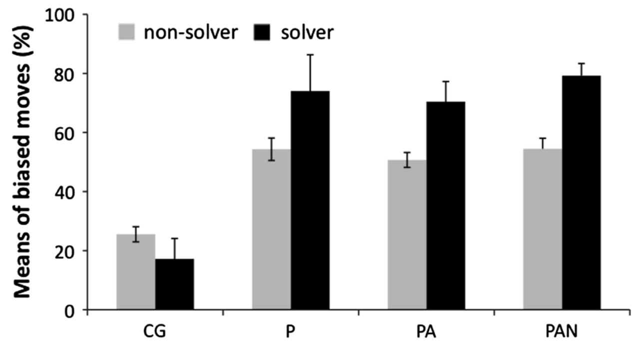

Fig. 4 Mean number of biased moves across groups and separated for solvers and non-solvers in comparison with the move pattern of the CG. Standard errors are plotted

As Fig. 4 illustrates, participants in all experimental conditions preferred moves that followed the direction of the arrows in the problem statement. Additionally, the figure demonstrates that solvers showed an even stronger preference for such moves than non-solvers.

An ANOVA with the between factors Condition (CG, P, PA, PAN) and Solver (non-solver, solver) and the dependent variable mean number of moves following arrows revealed a highly significant effect for the factor Condition, $F(3,\ 126) = 18.35$, $p < 0.01$, $\eta_p^2 = 0.30$. Post hoc comparisons (Scheffé) showed that all experimental conditions differed significantly from the CG condition ($p < 0.01$), and that the PAN condition differed from the P and the PA conditions ($p < 0.05$). There was no difference between the PA and P conditions. There was also a highly significant main effect for the factor Solver, $F(1,\ 126) = 10.52$, $p < 0.01$, $\eta_p^2 = 0.08$, with solvers' relying more strongly on arrow direction. There was no significant interaction ($p = 0.08$).

Solution rate

Table 1 provides an overview of the solution rate for each of the study conditions. The data showed that all experimental conditions showed a higher solution rate than the CG.

We analysed the influence of each additional piece of information (e.g. providing the path plus apex rather than just the path) on solution rates across the conditions using a

Table 1 Solution rates and non-dot moves classified according to different selection criteria

Condition	% and (#) of solvers	% and (#) of fast realization	% and (#) of Ps that made non-dot moves	Non-dot Ps who solve, % and (solvers/non-dot moves)
CG ($N = 34$)	11.76 (4)	5.88 (2)	23.53 (8)	50 (4/8)
P ($N = 34$)	26.47 (9)	14.71 (5)	29.41 (10)	90 (9/10)
PA ($N = 34$)	44.12 (15)	20.59 (7)	44.12 (15)	100 (15/15)
PAN ($N = 33$)	81.82 (27)	45.45 (15)	84.85 (28)	96 (27/28)

Table 2 Binary logistic regression model for the solution rates comparing all experimental conditions against the control group

Predictor	*B*	SE	Wald χ^2	*df*	Sig.	OR	95 % CI	
							Lower	Upper
Condition			29.75	3	0.01			
P	0.99	0.66	2.27	1	0.13	2.70	0.74	9.83
PA	1.78	0.64	7.86	1	0.01	5.92	1.71	20.54
PAN	3.52	0.70	25.43	1	0.01	33.75	8.60	132.53

SE standard error, *Sig.* significance (*p* value), *OR* odds ratio

binary logistic regression (BLR) (Hosmer & Lemeshow, 2000). BLR provides a method of analysing the influence of dichotomous, discrete, or continuous predictors on a binary outcome variable, and has already been successfully applied to the analysis of insight problem-solving experiments (e.g. Kershaw et al., 2013; Öllinger et al., 2012). BLR produces *B* values, and odds ratios. *B* values indicate the direction of the relationship; odds ratios indicate the likelihood that a participant in a particular group can be categorized as a solver, e.g. an odds ratio of 2 for a participant in a particular condition illustrates that the participant is 2 times more likely to solve the problem than for the baseline (CG) condition (see Kershaw et al., 2013).

Entering the three experimental conditions and the CG as baseline resulted in a significant model, $\chi^2(3, 135) = 40.61$ that classified 74.8 % of solvers correctly. Table 2 shows the BLR coefficient *B*, the Wald χ^2, and the odds ratio for each of the three conditions. The PA and PAN conditions differed significantly from the CG, the P condition did not. The regression coefficients were positive and increased with the amount of perceptual information provided. The odds ratio showed that a person in the PA group is 5.92 times more likely to solve the problem than a person in the CG. For the PAN group the odds ratio increases dramatically, to a value of 33.75.

Table 3 Binary logistic regression model for the fast realization data comparing all experimental conditions against the control group

Predictor	*B*	SE	Wald χ^2	*df*	Sig.	OR	95 % CI	
							Lower	Upper
Condition			14.34	3	0.01			
P	1.02	0.88	1.35	1	0.25	2.76	0.49	15.33
PA	1.42	0.84	2.85	1	0.09	4.15	0.79	21.66
PAN	2.59	0.81	10.27	1	0.01	13.33	2.73	65.02

SE standard error, *Sig.* significance (*p* value), *OR* odds ratio

Impact of perceptual constraints, pre-insight

We determined whether providing different degrees of perceptual information resulted in a faster realization of non-dot moves. We calculated the median number of moves until a non-dot move appeared across all solvers (Mdn = 4). Accordingly, we split the solvers into participants that had a fast or slow realization. Table 1 (column 3) illustrates that as expected, the number of participants that had a fast realization increased monotonically with the amount of perceptual information provided. We applied a BLR, using the CG as the reference category. The model was significant, $\chi^2(3, 135) = 16.82$, and classified 78.5 % of participants correctly. Table 3 shows the BLR coefficient *B*, the Wald χ^2, and the odds ratio for each of the three conditions. The data demonstrated that only the PAN condition differed significantly from the CG. The odds ratio indicates that participants in the PAN condition are 13.33 times more likely to overcome the boundary constraint within the first four moves than for the CG. For the P and PA conditions that do not provide non-dot locations, there were no statistical differences. This shows how strong the boundary constraint is in the nine-dot problem—participants will often fail to make use of arrows that point to locations outside of the perceived 3×3 square unless they are also accompanied by non-dot locations.

Impact of perceptual constraints, post-insight

We analysed whether the post-insight problem spaces were sufficiently constrained by the experimental conditions such that insight and problem solution were facilitated. In doing so, we determined how the experimental conditions facilitated problem solution once the boundary constraint was relaxed. Table 1 summarizes the data and shows that once insight has been achieved, the arrows play a major role in constraining the subsequent problem space. Almost all participants in the experimental conditions who made non-dot moves eventually solved the problem, whereas in the CG only half of the participants that drew at least one line to a non-dot position were able to solve the problem.

We tested whether the number of participants that made non-dot moves and solved the problem, and those who made non-dot moves and failed to solve the problem, varied between the conditions. With pairwise χ^2 tests we found that the PA condition differed from the CG, $\chi^2(1, 23) = 9.08$, $p < 0.01$, $\lambda = 0.50$ and the PAN differed from the CG, $\chi^2(1, 36) = 11.22$, $p < 0.01$, $\lambda = 0.38$. The P condition only marginally differed from the CG, $\chi^2(1, 18) = 3.55$, $p = 0.06$, $\lambda = 0.38$. There were no significant differences between the experimental conditions (ps > 0.20).

Discussion

The current study aimed to determine the role of problem space, search, impasse, and representational change on solution rates to the nine-dot problem. Our cognitive model hypothesized that insight problems are influenced by the size of the pre-insight search space, the difficulty of overcoming the impasse itself, and the size of the post-insight search space. We tested key components of our hypothesis by examining how the size of the search space influenced insight problem solving using the nine-dot problem. The pre-insight search space was constrained using arrows and additional perceptual information (see also Kaplan & Simon, 1990; MacGregor et al., 2001; Ormerod et al., 2002) to increase the likelihood of realizing that the applied solution strategies failed. Our cognitive model suggests that this in turn should facilitate impasse with the consequence that the likelihood of representational change increases (Ohlsson, 1992). Following the representational change, the size of the nine-dot problem space increases dramatically. The perceptual information provided therefore constrains this problem space, thus making problem solution more tractable. The results supported the predictions of the cognitive model, as we now discuss in detail.

Achieving insight

The only condition to benefit from perceptual information—in terms of overcoming the boundary constraint—was the PAN condition. The combination of arrows that indicate the solution path/apex and the additional information of a strong explicit perceptual hint to apply moves to non-dot locations clearly helps to overcome the boundary constraint more effectively than in all other conditions. 28 of 33 participants (85 %) overcame the constraint in the PAN condition, almost twice as many participants than in any other condition. The odds ratio indicates that in the PAN condition it was 13.33 more likely to overcome the boundary constraint than in the control group. This finding is important because as Chronicle et al. (2001) clearly showed, the nine-dot problem is resistant to additional perceptual information concerning non-dot locations, and supports Kershaw and Ohlsson's (2004) assumption that realizing the necessity of non-dot turns is a main source of problem difficulty. The provided arrows help to easily realize that a change of direction is necessary at the highlighted non-dot location. This is further supported by the number of participants whose first four moves include one that is outside of the perceived 3×3 square: 45 % of participants achieve this for the PAN, more than twice as many as any other condition. The P and PA conditions showed no significant facilitation. Thus, without an explicit indication that the solution requires moves to non-dot locations, the given path information is not helpful in overcoming the boundary constraint.

Representational change and solving the nine-dot problem

For the solution rate data we found that the PAN and the PA conditions differed from the control group. The PA condition provides an indication of the solution path (as per the P condition), but also additional path information related to the particular spatial pattern of the solution trajectory by providing the apex point where three lines meet. The additional apex information increases the odds ratio for a participant to be in the solver category from 2.7 in the P condition to 5.9 in the PA condition. Given the fact that there were no differences between the P and PA groups in overcoming the boundary constraint, it seems likely that group differences arise from the additional apex information in PA influencing the post-insight problem space.

However, the critical data that illustrates the problem of navigating the post-insight search space concerns the solution rates of the participants who overcame the boundary constraint. These 'conversion rate' data are quite remarkable (final column of Table 1). Once insight has been achieved, >90 % of participants are able to 'convert' their insight into a solution for the nine-dot problem in all of the perceptual hint conditions (P, PA, PAN), compared to 50 % in the control group. After a representational change, successful problem solvers still need to restrict the overly large problem space and this is what the perceptual hints help them to do. This finding can explain why previous studies found only small effects of hints that provided the information that participants have to draw lines outside the virtual boundaries either by verbal instruction or by visual cues (Burnham & Davis, 1969; Chronicle et al., 2001; Weisberg & Alba, 1981). That is, enabling participants to overcome the boundary constraint may not be beneficial to many participants who now have to navigate an even larger search space. As we have seen this is a complicated task that requires drawing lines to non-dot points, making turns in different directions, and configuring the remaining lines in an appropriate way (MacGregor et al., 2001).

Achieving 100 % success in the nine-dot problem

Even though we believe that the experimental conditions provided all of the necessary information to solve the nine-dot problem, approximately 20 % of the participants in the PAN condition were still unable to do so. A remaining challenge for participants in the PAN condition was to *combine* (according to Kershaw & Ohlsson, 2004) the solution path hints given by the arrows with the hints given by the non-dot locations. Clearly integrating the two different hints is an additional source of difficulty that some participants were unable to master.

A closer inspection of the non-dot data of the PAN condition showed that one non-solver made a total of 19 non-dot moves but was not able to combine the four lines in a way to find the solution (i.e. the participant was unable to combine the information given by the arrows to that given by the non-dot locations). The remaining four non-solvers in the PAN condition did not perform a single non-dot move (i.e. they were unable to use the arrow and non-dot information to relax the boundary constraint). Apparently, these participants suffered from a strong self-imposed constraint not to move outside the given nine dots. This again speaks for the power of a perceptually induced constraint. It also raises an important problem of our manipulation. The additional information given naturally changes the nature of the nine-dot problem (see Chronicle et al., 2001). The arrows can facilitate but also distract the problem-solving process, when participants have no idea how the provided perceptual information interplays with the task requirements.

There are a few aspects of our study that extend the knowledge about the nine-dot problem, and about insight problem solving in general. Important is the insight that a representational change is necessary *but not necessarily sufficient* for solving the problem; furthermore, while restricting the search space in an appropriate way is important to facilitate impasse, it is even more important after a representational change than prior. Combining our findings with those of relevant previous research, the model in Fig. 5 summarizes the cognitive processes we think are required to solve the nine-dot problem.

The model

The new aspect of our model is that we bring existing frameworks together and explicitly test the search dynamics before and after an impasse. The model merges theories of search (Kaplan & Simon, 1990; MacGregor et al., 2001) and of representational change. Ohlsson (1992; see also Ohlsson, 1984b) had already presented the different stages, but provided no experimental work that tested his assumption before and after an impasse. Moreover, a new aspect is the focus on the constraining of the search space, relaxing constraints, and again constraining

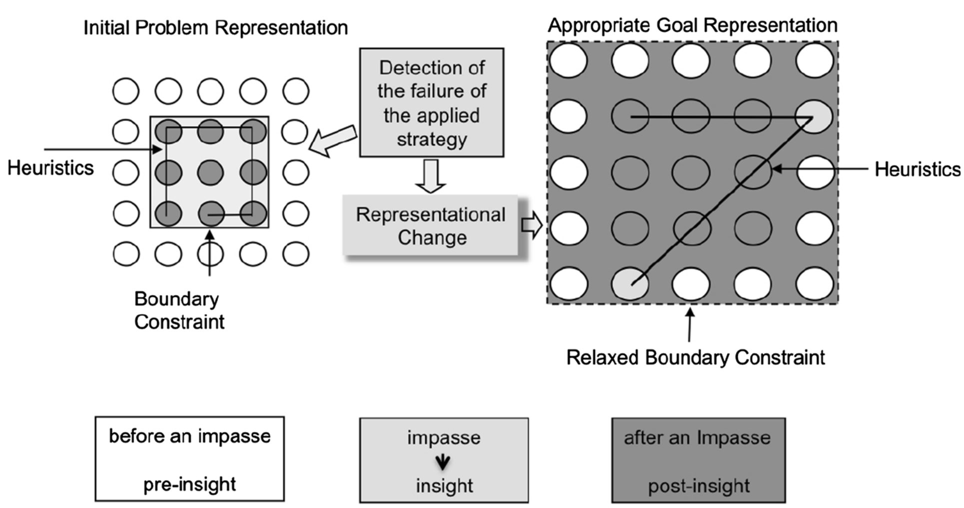

Fig. 5 The different cognitive processes that are needed to solve the nine-dot problem. Before an impasse, participants apply strategies such as hill-climbing in their attempts at solving the nine-dot problem (MacGregor et al., 2001); that is, they attempt to connect as many dots as possible with each move. After repeatedly failing to solve the problem using standard problem-solving heuristics the problem solvers reach impasse, because the heuristics do not help to change the problem representation per se as our data clearly demonstrated. After an impasse it is crucial to have the appropriate heuristic to restrict the now even larger search space. One has to consider that heuristics help to navigate through a problem representation, but if the representation is not appropriate for attaining the goal then the heuristic is useless

dynamics (see Fig. 1). That is, a circular restriction and expansion dynamic that provides the search space to be explored.

For the nine-dot problem, our model provides clarity to the varied and sometimes inconsistent results encountered in the literature. Of most importance here are two key points: first, an explicit instruction to draw lines that go beyond the perceived 3×3 grid (e.g. Burnham & Davis, 1969) will fail to dramatically increase solution rates because the resulting unconstrained search space is too large to be navigated effectively without any further hints; second, explicitly providing non-dot locations that are not part of the problem (i.e. as per Ormerod et al., 1997; see Fig. 2c) will not dramatically increase solution rates because non-dots alone are insufficient in generating a significant increase in relaxation of the "stubborn" boundary constraint. By providing arrows that restrict the direction, position and number of turns, the solution rates increase dramatically.

Our study adds to previous evidence demonstrating that a representational change is necessary for the solution to the nine-dot problem (Kershaw & Ohlsson, 2004). It adds to previous findings that a solution to the nine-dot problem can be effectively cued through providing solution path information together with cues that encourage people to cross the virtual boundary of the square formed by the dots. Importantly, the findings demonstrate that, while navigation of the problem space is important before a representational change, it is crucial after a representational change. Although heuristics play an important role before impasse, it is also necessary to adequately focus the search space after the insight has occurred.

We assume that the model can be also applied to other problems, and it should allow clear predictions as to how the search space before and after a potential impasse can be constrained in order to increase (1) the number of participants who encounter impasse; and (2) the number of participants who subsequently achieve insight and find the solution. For many insight problems, such as the eight-coin problem (Öllinger et al. 2012; Ormerod et al. 2002), problem solution becomes trivial once the insight has been realized and therefore key to such problems are the pre-impasse search heuristics and how quickly they enable impasse to be encountered. These have already been shown to influence impasse and subsequent solution rates for different initial configurations of the eight-coin problem (Ormerod et al. 2002). Of more interest are insight problems where solution to the problem is not so trivial once insight is realized. One such example is the four-tree problem outlined by Kershaw et al. (2013). The task is to arrange four trees such that each tree is located equidistant from each other. The model would predict that the problem is difficult because the initial search space is relatively unconstrained because there are unlimited configurations of four objects. That is, it is unlikely or time consuming for a problem solver to encounter impasse. A first restriction of the initial search space as Kershaw and colleagues showed is to provide conceptual information, e.g. the diagonal of a square is longer than the sides of the square. As a consequence, square solutions can be removed from the search space, increasing the likelihood that a problem solver encounters impasse. Additionally, the 3D constraint has to be relaxed (i.e. that problem solvers can consider three dimensions rather than two). After relaxing the 3D constraint a vast search space results. Thus, again, conceptual knowledge will be required to restrict the search space (e.g. a tetrahedron). As in the nine-dot problem, both conceptual and 3D information both have to be taken into account in order to successfully solve the problem.

Limitations

There are some limitations to our current study. First, the stimuli and arrow information used may imply confounding information.[2] The arrows indicate not only the direction of the solution sequence, but also that there are diagonal lines and the position of the turns. Providing diagonal lines can facilitate the solution as Lung and Dominowski (1985) demonstrated. However, all experimental conditions provided the diagonal information, and the perceptual conditions showed only a marginal increase of the solution rate. Second, the arrows also provide information that turns (particularly non-dot turns) are necessary for a solution. Our findings point out that the arrow information was particularly sufficient when the position of the non-dot points was given (PAN). That is, we provided in this condition at the same time the information of non-dot turns and non-dot points, and that two non-dot turns are necessary, a factor that plays, according to Kershaw and Ohlsson (2004), an important role. Further work will be necessary to disentangle these aspects.

In summary, our data provide strong evidence for multiple sources of problem difficulty supporting the findings of Kershaw and Ohlsson (2004), illustrating that the two key sources of difficulty in the nine-dot problem are the problem space (the pre-insight space is too restricted, the post-insight space becomes too large) and the required representational change.

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[2] This point was raised by an anonymous reviewer.

References

Ash, I. K., & Wiley, J. (2006). The nature of restructuring in insight: an individual-differences approach. *Psychonomic Bulletin & Review, 13*, 66–73.

Burnham, C., & Davis, K. (1969). The nine-dot problem: beyond perceptual organization. *Psychonomic Science, 17*, 321–323.

Chronicle, E. P., Ormerod, T. C., & MacGregor, J. N. (2001). When insight just won't come: the failure of visual cues in the nine-dot problem. *The Quarterly Journal of Experimental Psychology, 54A*, 903–919.

Duncker, K. (1945). *On Problem-Solving* (Vol. 58). Washington: American Psychological Association Inc.

Grant, E. R., & Spivey, M. J. (2003). Eye movements and problem solving: guiding attention guides thought. *Psychological Science, 14*, 462–466.

Hosmer, D. W., & Lemeshow, S. (2000). *Applied logistic regression*. New York: Wiley-Interscience.

Jones, G. (2003). Testing two cognitive theories of insight. *Journal of Experimental Psychology. Learning, Memory, and Cognition, 29*, 1017–1027.

Kaplan, C. A., & Simon, H. A. (1990). In search of insight. *Cognitive Psychology, 22*, 374–419.

Kershaw, T. C., Flynn, C. K., & Gordon, L. T. (2013). Multiple paths to transfer and constraint relaxation in insight problem solving. *Thinking & Reasoning, 19*, 96–136.

Kershaw, T. C., & Ohlsson, S. (2004). Multiple causes of difficulty in insight: the case of the nine-dot problem. *Journal of Experimental Psychology. Learning, Memory, and Cognition, 30*, 3–13.

Knoblich, G., Ohlsson, S., Haider, H., & Rhenius, D. (1999). Constraint relaxation and chunk decomposition in insight problem solving. *Journal of Experimental Psychology. Learning, Memory, and Cognition, 25*, 1534–1555.

Litchfield, D., & Ball, L. J. (2011). Using another's gaze as an explicit aid to insight problem solving. *The Quarterly Journal of Experimental Psychology, 64*, 649–656.

Lung, C.-T., & Dominowski, R. L. (1985). Effects of strategy instructions and practice on nine-dot problem solving. *Journal of Experimental Psychology. Learning, Memory, and Cognition, 11*, 804–811.

MacGregor, J. N., Ormerod, T. C., & Chronicle, E. P. (2001). Information processing and insight: a process model of performance on the nine-dot and related problems. *Journal of Experimental Psychology. Learning, Memory, and Cognition, 27*, 176–201.

Maier, N. R. F. (1930). Reasoning in humans. I On direction. *Journal of Comparative Psychology, 10*, 115–143.

Newell, A., & Simon, H. A. (1972). *Human problem solving*. Englewood Cliffs, NJ: Prentice Hall.

Ohlsson, S. (1984a). Restructuring revisited: I. Summary and critique of the Gestalt theory of problem solving. *Scandinavian Journal of Psychology, 25*, 65–78.

Ohlsson, S. (1984b). Restructuring revisited: II. An information processing theory of restructuring and insight. *Scandinavian Journal of Psychology, 25*, 117–129.

Ohlsson, S. (1992). Information-processing explanations of insight and related phenomena. In M. Keane & K. Gilhooly (Eds.), *Advances in the Psychology of Thinking* (pp. 1–44). London: Harvester-Wheatsheaf.

Öllinger, M., Jones, G., Faber, A. H., & Knoblich, G. (2012). Cognitive mechanisms of insight: the role of heuristics and representational change in solving the eight-coin problem. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 39*(3), 931–939.

Öllinger, M., Jones, G., & Knoblich, G. (2008). Investigating the Effect of Mental Set on Insight Problem Solving. *Experimental Psychology, 55*, 269–282.

Ormerod, T. C., Chronicle, E. P., & MacGregor, J. N. (1997). *Facilitation in variants of the nine-dot problem: perceptual or cognitive mediation? Poster presented at the 18th Annual Conference of the Cognitive Science Society*. Stanford: CA.

Ormerod, T. C., MacGregor, J. N., & Chronicle, E. P. (2002). Dynamics and constraints in insight problem solving. *Journal of Experimental Psychology. Learning, Memory, and Cognition, 28*, 791–799.

Scheerer, M. (1963). Problem-Solving. *Scientific American, 208*, 118–128.

Thevenot, C., & Oakhill, J. (2008). A generalization of the representational change theory from insight to non-insight problems: the case of arithmetic word problems. *Acta Psychologica, 129*, 315–324.

Thomas, L. E., & Lleras, A. (2007). Moving eyes and moving thought: on the spatial compatibility between eye movements and cognition. *Psychonomic Bulletin & Review, 14*, 663–668.

Weisberg, R. W., & Alba, J. W. (1981). An examination of the alleged role of "fixation" in the solution of several "insight" problems. *Journal of Experimental Psychology: General, 110*, 169–192.